INTERNATIONAL DESIGN CONFERENCE - DESIGN 2002
Dubrovnik, May 14 - 17, 2002.

# DAMAGE TOLERANT DESIGN OF THE AIRCRAFT COMPONENTS

Damir Jelaska, Srdjan Podrug, Mirko Grljušic and Srecko Glodež

*Keywords: Crack initiation, reshaping crack growth rate formulae, Block crossing effect*

## 1. Introduction

Any machine part subjected to substantial load due to start-stop operations has basically a similar stress history consisting of $N_B$ stress blocks (one for each operation) with $n_{HCF}$ high cycle fatigue (HCF) cycles and one low cycle (LCF) cycle (Fig. 1). LCF stresses are actually the "steady" stresses, which result in one cycle for every start-up and shutdown operation [1], and HCF stresses are caused by in-service vibrations. The integrity of the parts of high-speed engines, especially the turbine and compressor discs and blades is particularly critical, because the usually extremely high cyclic frequencies of in-service loading spectra cause that the fatigue life of e.g. $10^7$ cycles can be reached in a few hours. It was one of the reasons that a number of fatigue failures has been detected e.g. in US fighter engines [Nicholas and Zuiker 1996]. It is important therefore, to keep looking for a simple procedure enabling designer the reliable estimation of both crack initiation and crack propagation life for a given applied load, or to obtain the (boundary) load (or strain), at which the component would not experience the unpermissible damage during the designed life. The damage tolerant design normally refers to the design methodology in which fracture mechanics analyses predict remaining life and quantify inspection intervals. That philosophy allows the flows to remain in the structure, provided they are well below the critical size. Among the significant learned papers treating this matter, there is no one taking into account the additional damage when crossing from HCF stress block to LCF one, or reversely. That is the one more reason for this paper.

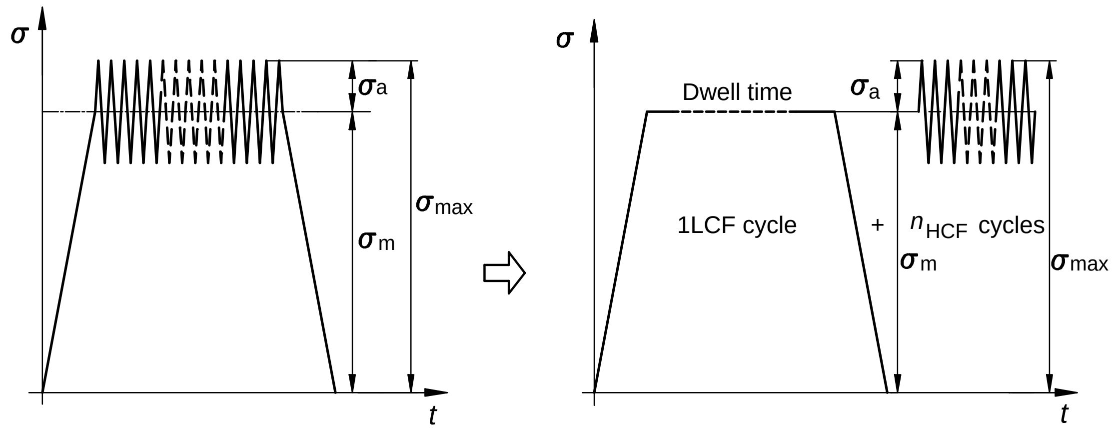



**Figure 1. Common stress history of one combined stress block and its separation in one LCF stress cycle and one HCF stress block**

## 2. Crack initiation assessment

### 2.1 Crack initiation life at HCF loading

The *S-N* curve for crack initiation is described by the Wöhler type equation [Nicholas and Zuiker 1996, Singh 2002)]

$$N_i s^{m_i} = C_i \tag{1}$$

where Ni is the crack initiation life for a certain stress level s, and mi and Ci are the material constants.
At steady loading (N = 1/4), the CI curve equals ultimate strength s U, and for the sufficiently long fatigue life, which can be taken as e.g. Ngr, it equals the endurance limit s 0, which mean the entire fatigue life at the endurance limit level consists of the crack initiation life. On the basis of assumption that there is a unique CI curve between these two points, its slope was approximated [Jelaska 2000] as

$$m_i = \frac{\log(4N_{gr})}{\log(s_U / s_0)} \tag{2}$$

This expresion was found to be in good correllation with experimentaly obtained values. For example, the fatigue strength exponent b of steel 42 Cr Mo 4V (after DIN) for initiation life at r = -1 loading, was found to equal 0,0692 [Grubisic and Sonsino 1982], thus mi = 1/b = 14,5. Exactly the same value was obtained after Eqn. (2) for Ngr = 3·107. It is also in line with novel investigations of [Singh 2002]. Whereas at the endurance limit stress level the initiation life practically equals the total fatigue life, the constant Ci can be assessed as Ci=Ngr $s_0^{m_i}$ , where Ngr is the number of cycles at the knee of the S-N curve.
For the purpose of this paper, the French curve at r = 0 is used, which enables determining the level of the pulsating stress at the CI boundary for certain Ni, by knowing the crack initiation life Ngr at the endurance limit level:

$$s_{0N,i} = s_0 \left(k_{gr} N_{gr} / N_i\right)^{1/m_i} \tag{3}$$

### 2.2 Crack initiation life at combined HCF/LCF loading

For the stress history described in Fig. 1., the crack initiation life expressed in number of stress blocks $N_{B,i}$, is derived on the basis of Palmgren - Miner hypothesis of linear damage accumulation, where the level of damage is defined as

$$D_i = \sum_{j=1}^{n_B} \frac{n_j}{N_j} = \sum_{j=1}^{n_B} \left( \frac{n_{HCF,j}}{N_{HCF,j}} + \frac{1}{N_{LCF,j}} \right) \tag{4}$$

The CPT is reached for $D_i = 1$, when number of blocks $n_B$ becomes $N_{B,i}$. It is easy then to determine the crack initiation life expressed in stress blocks [Nicholas and Zuiker 1996]:

$$N_{B,i} = \frac{1}{\frac{n_{HCF}}{N_{HCF,i}} + \frac{1}{N_{LCF,i}}} \tag{5}$$

The total initiation life is

$$N_i = N_{B,i}(1 + n_{HCF}) \cong N_{B,i} \cdot n_{HCF} = \frac{1}{\frac{1}{N_{HCF,i}} + \frac{1}{N_{LCF,i} \cdot n_{HCF}}} \tag{6}$$

The initiation life NLCF,i is determined after the CI curve (3) at r = 0:

$$N_{LCF} = N_{gr}\left(s_0 / s_m\right)^{m_i} \tag{7}$$

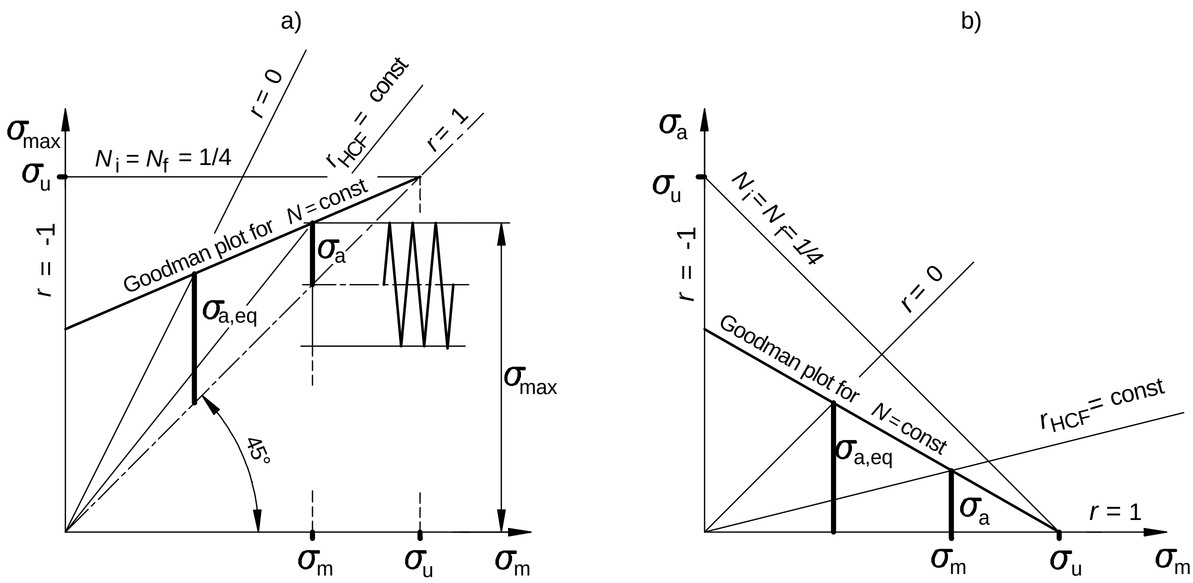



**Figure 2. Reducing the HCF stress amplitude s$_{\mathbf{a}}$ to an equivalent stress amplitude s$_{\mathbf{a,eq}}$ at r = 0 in a) Smith diagram, and b) Haigh diagram**

Since the Palmgren-Miner hypothesis is valid for various stress blocks at the same stress ratio, this equation is also used for the calculation of the HCF initiation life, but by substituting in it an equivalent stress range obtained by reducing a HCF stress range (with stress ratio $r_{HCF} > 0$) to an equivalent stress range at $r = 0$ (Fig. 2). This equivalent stress range is obtained as intersection point of Goodman plot $N_i$ =const having the slope $(s_U - s_{\max})/(s_U - s_m)$ and load line at $r$=0. It is obtained:

$$\Delta s_{eq} = 2 s_{a,eq} = \frac{2 s_a s_U}{s_U + s_a - s_m} \tag{8}$$

Thus, by substituting Eqn. 7 in Eqn. 6 twice (for a LCF stress $s_m$ , and for a reduced HCF stress after Eqn. 8), the explicit formula is obtained for determining the crack initiation life at combined HCF/LCF loading:

$$N_i = \frac{N_{gr} s_0^{m_i}}{\left(\frac{2s_U s_a}{s_U + s_a - s_m}\right)^{m_i} + \frac{s_m^{m_i}}{n_{HCF}}} \tag{9}$$

## 3. Crack propagation assessment for combined hcf/lcf loading

### 3.1 Reshaping the crack growth rate formulae

The fatigue crack growth rate formulae valid in regions II and III of crack growth rate [Forman 1967, Ritchie et al. 1999, etc.], and therefore acceptable for the estimation of the crack propagation life at constant amplitude loading, can be generally noted down as

$$\frac{da}{dN} = f(\Delta K^m, K_{\max}^n) \tag{10}$$

where

$$\Delta K = \Delta s Y \sqrt{pa} \tag{11}$$

is the stress intensity range,

$$K_{\max} = s_{\max} Y \sqrt{pa} \tag{12}$$

is the upper value of the stress intensity factor, m and n are material constants, $\Delta s = 2s_a$ is a stress range, $s_{\max}$ is a maximum stress, Y is a crack form factor, and a is a crack size.

By introducing into the formula (9) the damage ratio $D = a/a_c$ , where ac is a critical crack size and fracture toughness $K_c = s_{\max} Y \sqrt{p\, a_c}$ , it can be reshaped in the form

$$\frac{dD}{dN} = \frac{1}{a_c} f_1\left[(\Delta K/K_c)^m, (K_{\max}/K_c)^n\right] = \frac{D_0}{a_0} f_1\left[\left(2(1-r)D^{\frac{1}{2}}\right)^m, D^{\frac{n}{2}}\right] = f_2(D) \tag{13}$$

where $a_0$ is initial crack size, $D_0 = a_0/a_c$ is an initial damage ratio and $r = s_{\min}/s_{\max} = K_{\min}/K_{\max}$ is a load (stress intensity) ratio. By integrating this formula, it is easy to obtaine the damage ratio after *N* propagating cycles and to determine the crack propagation life at constant amplitude loading - by substituting in it *D*= 1. The equation (14) can be used also in fatigue assessments at variable amplitude loading [Pavlov 1988], but in such a case $a_c$ changes, if $s_{\max}$ changes. So,Eqn. (13) must be reshaped:

$$\frac{dD}{dN} = \frac{1}{a_c}\frac{da}{dN} - \frac{a}{a_c^2}\frac{da_c}{dN} = f_2(D) + \frac{D}{D_0}\frac{dD_0}{dN} \tag{14}$$

In the case of block loading, or if the spectrum loading is approximated with block loading, the second term of this equation always equals zero, except when crossing from one stress block to another- just when the first term becomes zero.
During the change of $a_c$, the equation (14) can be written in the form

$$\frac{dD}{D} = -\frac{da_c}{a_c} \tag{15}$$

By integrating it, the increased value of damage ratio caused by the change of the critical crack size between two stress blocks, is obtained:

$$D_2 = D_1 \frac{a_{c1}}{a_{c2}} \tag{16}$$

The expression (14) is appropriate for the crack propagation assessment at any loading conditions, including non-regular one, where maximum stress, crack form factor and load ratio change.

### 3.2 Explicit expression for approximate estimation of the crack propagation life at combined HCF/LCF loading

Herein, the formula (14) is applied for the crack propagation life estimation in the gas turbine and compressor discs and blades made of the titanium alloy Ti-6Al-4V, at combined HCF/LCF loading. If the stress history is simplified in the way that it consists of one LCF stress block with $N_{LCF} = N_B$ cycles at maximum stress $s_m$ and load ratio $r = 0$, followed by one HCF stress block with $n_{HCF} \cdot N_B$ cycles at maximum stress $s_{max}$ and load ratio $r = (s_{max} - 2 s_a)/ s_{max}$, then the initial damage ratio is

$$D_0 = pa_0 \left( \frac{Y_{Lc} s_m}{K_c} \right)^2 \tag{17}$$

where $Y_{Lc}$ is a crack form factor at $K_c$ stress intensity of the LCF loading. After Raju and Newman [Raju and Newman 1986], the form factor is approximated by

$$Y = 0{,}78 \left( 1 + \frac{a}{d} \right) \tag{18}$$

where $d$ is a bar diameter.
As most appropriate for the purpose of this paper, the Ritchie formula [Ritchie et al. 1999] for the crack growth rate

$$\frac{da}{dN} = C\Delta K^m K_{\max}^n \tag{19}$$

is applied for determining the damage ratio. For titanium alloy Ti-6Al-4V, the following values of material constants were obtained: $C$=5,2·10$^{-12}$ , $m$=2,5 and $n$=0,67. The damage ratio growth rate is obtained

$$\frac{dD}{dN} = \frac{B}{a_c}(1-r)^m D^{\frac{m+n}{2}} \tag{20}$$

where $B = 2^m C K_c^{m+n}$ is a material constant. By integrating this equation, it is easy now to determine the damage ratio at the end of LCF stress block:

$$D_{LCF} = \frac{D_0}{\left[1 - D_0^{\frac{m+n}{2}-1} \frac{B}{2a_{cL}}(m+n-2)N_B\right]^{\frac{2}{m+n-2}}} \tag{21}$$

According to (16), at the beginning of the HCF stress block, the damage ratio is

$$D_{0,HCF} = D_{LCF}\frac{a_{cL}}{a_{cH}} \tag{22}$$

where $a_{cL}$ and $a_{cH}$ are the critical values of the srack size at LCF and HCF loading, respectively. Those values can be determined by solving their equations. E.g. $a_{cH}$ is determined from the equation

$$a_{cH} = \frac{1}{p}\left[\frac{K_c}{Y(a_{cH})s_{\max}}\right]^2 \tag{23}$$

where $K_c$=50 MPa m$^{1/2}$ for Ti-6Al-4V alloy, [Ritchie et al. 1999].
The damage ratio at the end of the HCF stress block, as the final damage ratio, is now

$$D_{HCF} = \frac{D_{0,HCF}}{\left[1 - D_{0,HCF}^{\frac{m+n-2}{2}} \frac{B}{2a_{cH}}(1-r)^m (m+n-2) n_{HCF} N_B\right]^{\frac{2}{m+n-2}}} \tag{24}$$

The fatigue fracture occurs when this damage ratio reaches the value of one. Then, from the equations (21),(22) and (24), it is not difficult to solve for the $N_B$ and consequently for the entire crack propagation life:

$$N_p = 2\frac{\left(a_0/a_{cH}\right)^{1-\frac{m+n}{2}} - 1}{B(m+n-2)\left[(1-r)^m n_{HCF}/a_{cH} + a_{cL}^{-1}\left(a_{cL}/a_{cH}\right)^{1-\frac{m+n}{2}}\right]} n_{HCF} \tag{25}$$

Thus, the explicit expression is derived, enabling the estimation of the crack propagation life at combined HCF/LCF loading, for certain values of the stress levels $s_{\max}$ and $s_m$, which are hidden in $a_{cH}$ and $a_{cL}$.
When no "block crossing" effect is applied, the expression for the crack propagation life becomes

$$N_p = 2\frac{D_0^{1-\frac{m+n}{2}} - 1}{B(m+n-2)\left[(1-r)^m n_{HCF}/a_{cH} + a_{cL}^{-1}\right]} n_{HCF}. \tag{26}$$

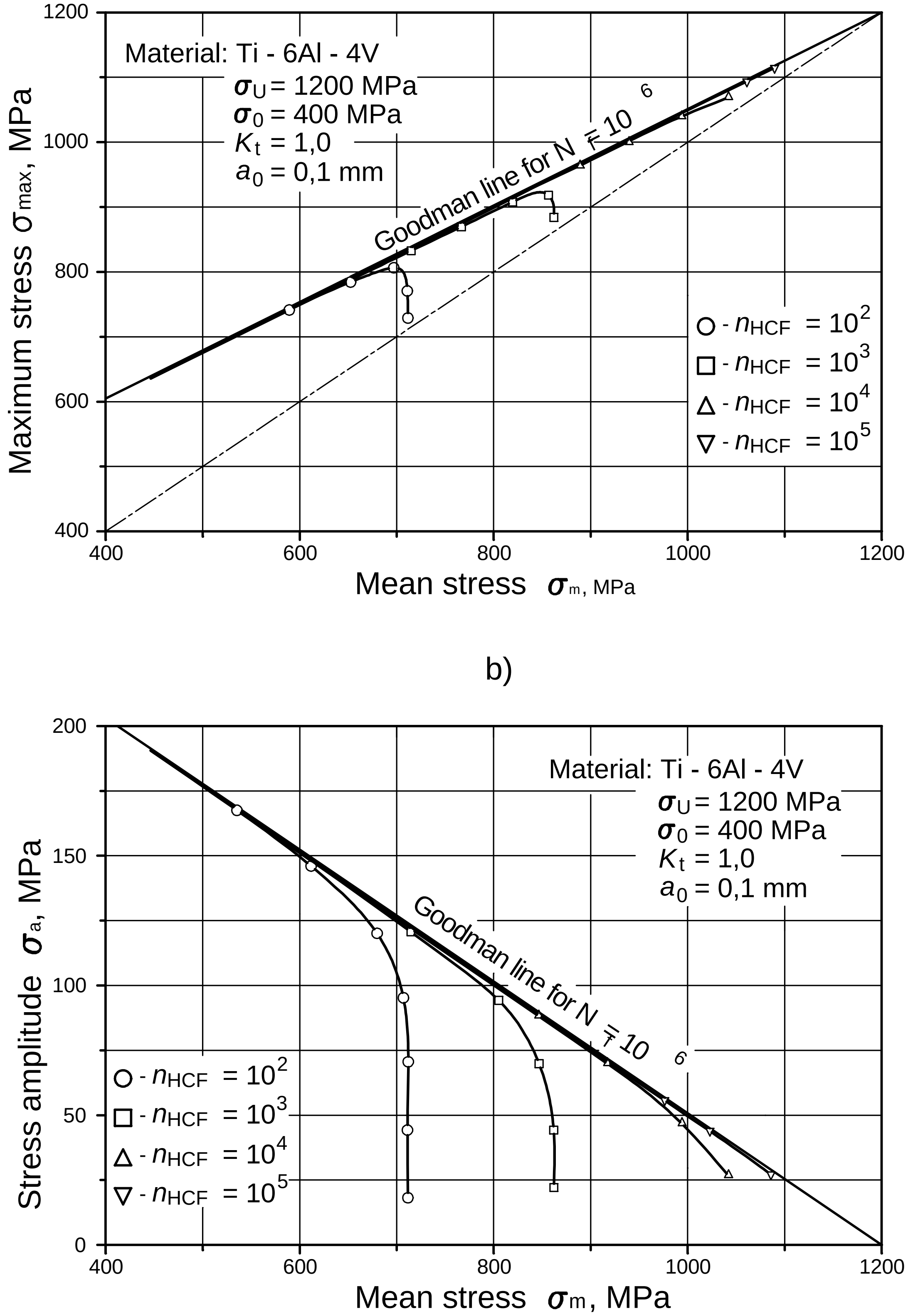
### 3.3 A more precise procedure for the crack propagation assessment at combined HCF/LCF loading

Assumption that stress history consists of one LCF cycle followed by one HCF stress block consisting of $n_{HCF}$ cycles, followed by one LCF cycle etc. (Fig. 1) is much closer to real operational conditions. Thus, after one LCF cycle, the damage ratio is obtained by substituting in Eqn. (21) $N_B$=1. At the beginning of the HCF stress block, the damage ratio is obtained according to (16), and at the end of the HCF stress block, the damage ratio is obtained by substituting in (24) $N_B$=1. This damage ratio decreased after (16) is an initial damage ratio at the beginning of the second combined stress block, etc. The fatigue fracture occurs at the moment when damage ratio reaches the value of one. Then, the reached life becomes the fatigue life for certain, input values of stress levels $s_{\max}$ and $s_m$. For the subject material, Ti-6Al-4V, the fatigue lives determined in such a way, were compared with those obtained in "regular way", not taking into account the block crossing effect. No signifficant differences in fatigue lives are obtained between these two procedures. There is also no signifficant difference between these fatigue lives and those obtained after formulae (25) and (26). It means that "block crossing" effect does not play a role in the simple combined HCF/LCF loading presented here.

## 4. Fatigue limits for combined hcf/lcf loading

In fatigue design generally, and especcially in design of components subjected to combined HCF/LCF loading, the Smith (or Haigh) diagram is a very useful tool, presenting the areas, i.e. the stress levels at which the required fatigue life will not be reached. The corresponding curves obtained, enable damage tolerant design, i.e. they divide the diagram area in two zones: the zone of stress states resulting in allowable and unallowable fatigue lives, that is in allowable and unallowable damage level. The procedure is the same as described in previous chapter, but for the fatigue life as input data. Thus, for certain values of fatigue lives, the fatigue strength curves are obtained indicating the stress levels in Smith diagram causing the fatigue failure after $N_f = C_f$ cycles. The calculations are carried out for various values of $C_f$, and for a number of HCF cycles per one stress block $n_{HCF} = 10^2 \ldots 10^5$. The fatigue limit curves obtained precisily exibit the reduction of the design area in the Smith diagram compared to HCF loading only, the more so as the share of LCF loading is greater.
As an example, the resulting $N_f$=$10^6$ curves for titanium alloy Ti-6Al-4V, and for $n_{HCF} = 10^2 \ldots 10^5$, are exhibited in Smith and Haigh diagrams, Fig. 3. In view of these curves, which share the diagram space on the safe and the unsafe one, it is observed:

- These curves are located in smith diagram between Goodman line and σmax = σm straight line, the higher the nHCF the higher the curve position. At the region of lower mean stresses, they make one with Goodman line, then separate from it, reach maximum, and finaly fall down to the constant mean stresses. Thus, the presence of the LCF component restricts the a)




b)



**Figure 3. Fatigue strengths in Smith (a) and Haigh (b) diagram for a combined HCF/LCF loading of a titanium alloy Ti-6Al-4V**

safe design space compared to that in case of pure HCF, the more so as the share of the LCF component is greater.

- The block crossing effect does not influence significantly the curves of constant fatigue life.
- Between the curves of constant fatigue life based on initial crack sizes of 0,1 mm and 0,05 mm was not observed a significant difference.
- The curves of constant fatigue life obtained on the basis of the derived closed form fatigue life formula, and those obtained on the basis of growth increments computed for one LCF cycle, nHCF cycles, next LCF cycle, etc., do not differ significantly.

## 5. Summary and conclusions

The closed form expression for estimation of the crack initiation life at combined HCF/LCF loading is derived in this paper, and the way of reshaping the crack growth rate formulae in the form enabling their use in fatigue design at non-stationary loading is demonstrated. This new derived formula suggests an additional damage increase when crossing from one stress block to another. So, fatigue design becomes more conservative, broaching the subject of reliability of recent fatigue assessment of the components under variable amplitude loading. Herein, the reshaped crack growth rate formula is applied for the fatigue design of aircraft components made of titanium alloy Ti-6Al-4V and subjected to combined HCF/LCF loading. For the stress history simplified in the way that it consists of one LCF stress block with $N_{LCF} = N_B$ cycles at maximum stress $s_m$ and load ratio $r = 0$, followed by one HCF stress block with $n_{HCF} \cdot N_B$ cycles at maximum stress $s_{max}$ and load ratio $r = (s_{max} - 2s_a)/s_{max}$, the closed form expression is derived for estimating the crack propagation life at combined HCF/LCF loading.

Smith and Haigh diagrams as design tools for estimating the fatigue strengths for designed fatigue life, known load ratio and various number $n_{HCF}$ cycles, are obtained and presented for the parts made of titanium alloy Ti-6Al-4V and subjected to combined HCF/LCF loading.The results of this research should be taken as a guide because

- The small crack behaviour has not been taken into account,
- The presence of other damage mechanisms like creep fatigue, oxidation and other environmental effects are ignored,
- The residual stresses have not been handled,
- The stress concentration has been ignored,
- Technology faults, material quality and operating conditions (like elevated temperature), have not been taken into account,
- Linear damage summation rule has been applied, although more precise techniques exist,
- The presence of inclusions and the service-induced damages could not be clasped in calculations,
- The reliability aspect of the design has been ignored.

At the same time, these imperfections are the sign posts in the direction of building an expert system for the fatigue design of the aircraft components subjected to combined HCF/LCF loading.

### References

*Buxbaum, O., "Betriebfestigkeit", Verlag Stahleisen, Düsseldorf, 1986.*

*Forman, R.G., Keary, V.E., and EngleR.M., "Numerical analyses of crack propagation in cyclic loaded structures", Journal of Basic Engineering, Vol. 89,1967,pp 459-464.*

*French, H. Trans. of ASTM, 21 (1933), p. 899.*

*Grubisic,V. Sonsino,C.M., "Influence of Local Strain Distribution on the Low Cycle Fatigue Behaviour of Thick-Walled Structures", in: ASTM STP 770 (1982), pp. 612 - 629.*

*Jelaska,D., "Crack initiation life at combined HCF/LCF load", In: Life assessment and management for structural components, Vol.I, pp.239-246, National Academy of Sciences of Ukraine, Kiev, 2000.*

*Nicholas, T.J.R. and J. R. Zuiker,J.R. "On the use of the Goodman diagram for high cycle fatigue design", Int. J. Fracture, 80, 1996, pp 219-235..*
*Pavlov, P.A.: "Fundamentals of Fatigue Design of Machine Parts", Mashinostroenie, Leningrad, Russia, 1988.*
*Raju, I. S.,Newman, J. C., in Fracture Mechanics: Seventeenth Volume, ASTM STP 905, Philadelphia 1986, pp 789-805.*
*Ritchie, R.O. et al.: High-cycle fatigue of Ti-6Al-4V, Fatigue Fract. Engng. Mater. Struct. 22,1999, pp 621-631.*
*Singh, A., "The nature of initiation and propagation S-N curves at and below the fatigue limit", Fatigue & Fract. Engng. Mater. & Structures, 25, 2002, pp 79 - 89.*

Damir Jelaska, Sc.D.
University of Split, FESB, Department of Mechanical Engineering
R. Boskovica b.b., Split, Croatia
Tel.: + 385 21 305874
Telefax: + 385 21 463877,
Email: djelaska@marjan.fesb.hr